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Palestinian National Authority Ministry of High Education Palestine Technical University Name: ..... 20911132	بسم الله الرحمن الرحيم 	Numerical Analysis Mon. 2/1/2012 Period: 1 Hour Second Exam B
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Question 1:

10-11

(10 points)

Circle the correct answer:

1) If  $p(x) = (x+1)^4(x-1)^3$  then 1 is a zero of multiplicity:

- a) 3   b) 4   c) 7   d) 1

2) The sequence  $\{2^{-3^n}\}_{n=1}^{\infty}$  converges to:

- a) zero linearly   b) zero cubically   c) zero quadratically   d) one linearly

3) Let  $p_0=2, p_1=2.1, p_2=2.2$  then  $\Delta^2 p_0 =$ :

- a) 0.1   b) 0.4   c) 0.2   d) 0

4) Let  $x_0=2, x_1=2.5$ , then the first interpolating polynomial of  $f(x)=x^2$  is:

- a)  $-30.5-27$    b)  $30.5x+27$    c)  $4.5x-5$    d)  $-4.5x+5$

Question 2:

(20 points)

Find an approximation " $x_2$ " to one of the zero's of  $p(x) = x^3 - x - 1$  using Newton-Raphson and synthetic division where  $x_0 = 1$  (using four digit rounding).

$p(x) = x^3 - x - 1$

$x^3 \quad x^2 \quad x^1 \quad x^0$

1   0   -1   -1

1

1   0   -1

-1

$R(1) = -1$

$p(1) = -1$

$x^2 \quad x^1 \quad x^0$

1   1   0

1

2   2

$R'(1) = 2$

$p'(1) = 2$

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⊛ by Newton Raphson Method :-

$$P_1 = P_0 - \frac{f(P_0)}{f'(P_0)}$$

$$\therefore P_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \left(\frac{-1}{2}\right) = 1 - -\frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2} = \boxed{1.5}$$

	$X^3$	$X^2$	$X^1$	$X^0$
	1	0	-1	-1
		⊕	⊕	⊕
1.5		1.5	2.25	1.875
	1	1.5	1.25	0.875

$\Rightarrow f(1.5) = \boxed{0.875}$

$\therefore f(1.5) = 0.875$

	$X^2$	$X^1$	$X^0$
	1	1.5	1.25
		⊕	⊕
1.5		1.5	<del>4.5</del>
	1	<del>3</del>	5.75

$\Rightarrow f'(1.5) = \boxed{5.75}$

$\therefore f'(1.5) = 5.75$

$$\therefore X_2 = X_1 - \frac{f(X_1)}{f'(X_1)} = 1.5 - \frac{f(1.5)}{f'(1.5)}$$

$$= 1.5 - \frac{0.875}{5.75}$$

$= \boxed{1.3478}$



b) Suppose that the Taylor series for a function is

$$f(x) = x + \frac{1}{2}x^3 + \frac{1}{3}x^5 + \frac{1}{4}x^7 + \dots$$

And  $g(x) = 1 - x^2 + x^4 - x^6 + \dots$

, consider  $F(x) = f(x) - xg(x)$ , find the multiplicity of the root  $x=0$ .

$$f(x) = x + \frac{1}{2}x^3 + \frac{1}{3}x^5 + \frac{1}{4}x^7 + \dots$$

$$g(x) = 1 - x^2 + x^4 - x^6 + \dots$$

$$xg(x) = x - x^3 + x^5 - x^7 + \dots$$

$$\begin{aligned} F(x) = f(x) - xg(x) &= \left(x + \frac{1}{2}x^3 + \frac{1}{3}x^5 + \frac{1}{4}x^7 + \dots\right) - \left(x - x^3 + x^5 - x^7 + \dots\right) \\ &= \frac{1}{2}x^3 + \frac{4}{3}x^5 - \frac{3}{4}x^7 + \dots \end{aligned}$$

$\Rightarrow$  Taylor series:  $1 + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \dots$

$$\frac{1}{2}x^3 = \frac{f'''(x_0)}{6}x^3$$

$$\frac{f'''(x_0)}{6} = \frac{1}{2} \Rightarrow f'''(x_0) = \frac{6}{2} = 3$$

$$f''(x_0) = 3x_0 + c \Rightarrow f'(x_0) = \frac{3x_0^2}{2} + c \Rightarrow f(x_0) = \frac{3x_0^3}{2 \cdot 3} = \frac{3x_0^3}{6}$$

$$f(x) = \boxed{\frac{x^3}{2}}$$

$$g(x) = 1 - x^2 = \frac{f''(x_0)}{2}x^2 \Rightarrow -1 = \frac{f''(x_0)}{2}$$

$$f''(x_0) = -2 \Rightarrow f'(x_0) = -2x_0 \Rightarrow f(x_0) = -\frac{2x_0^2}{2} = \boxed{-x_0^2}$$

$$xg(x) = x(-x^2) = -x^3$$

$$\therefore F(x) = f(x) - xg(x) = \frac{x^3}{2} - (-x^3) = \frac{x^3}{2} + x^3$$

$$= \boxed{\frac{3x^3}{2}}$$

$\therefore F(x)$  has a multiplicity of  $\boxed{3}$  at  $x=0$

Question 3:

(20 points)

The following table lists the values of  $f(x) = \sqrt{x}$  accurate to the places given,  $x \in [1.2, 1.6]$

	$x_i$	$F(x_i)$
$x_0$	1.2	1.095
$x_1$	1.4	1.183
$x_2$	1.6	1.265

- Use Nevill's method to approximate  $f(1.5)$ .
- Find the actual error.
- Find the upper bound of the error.

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$$Q_{00} = P_0 = f(x_0)$$

$$Q_{10} = P_1 = f(x_1)$$

$$Q_{20} = P_2 = f(x_2)$$

$$Q_{11} = P_{01} = \frac{(x - x_0)P_1 - (x - x_1)P_0}{x_1 - x_0}$$

$$Q_{11}(1.5) = \frac{(x - 1.2)(1.183) - (x - 1.4)(1.095)}{1.4 - 1.2}$$

$$= \frac{(1.5 - 1.2)(1.183) - (1.5 - 1.4)(1.095)}{0.2}$$

$$= \frac{(0.3)(1.183) - (0.1)(1.095)}{0.2} = \boxed{1.227}$$

$$Q_{21} = P_{12} = \frac{(x - x_1)P_2 - (x - x_2)P_1}{x_2 - x_1}$$

$$Q_{21}(1.5) = \frac{(1.5 - 1.4)(1.265) - (1.5 - 1.6)(1.183)}{1.6 - 1.4}$$

~~Good Luck~~

$$= \frac{(0.1)(1.265) - (-0.1)(1.183)}{0.2} = \boxed{1.224}$$

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$$Q_{22} = P_{012} = \frac{(x - x_0)P_{12} - (x - x_2)P_{01}}{x_2 - x_0}$$

$$= \frac{(1.5 - 1.2)(1.224) - (1.5 - 1.6)(1.227)}{1.6 - 1.2}$$

$$= \frac{(0.3)(1.224) + (0.1)(1.227)}{0.4} = 1.22475$$

b) Actual error =  $|f(1.5) - Q_{22}(1.5)|$

$$f(1.5) = \sqrt{1.5} = 1.22474$$

$$Q_{22}(1.5) = 1.22475$$

$$\Rightarrow \text{Actual error} = |1.22474 - 1.22475| = 0.00001 = 1 \times 10^{-5}$$

c) upper bound

$$= \left| \frac{f'''(z)}{3!} (x - x_0)(x - x_1)(x - x_2) \right|$$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{4} x^{-\frac{3}{2}}$$

$$f'''(x) = \frac{+3}{8} x^{-\frac{5}{2}} \Rightarrow f'''(x) = \frac{3}{8} x^{-\frac{5}{2}} = \frac{3}{8} \frac{1}{\sqrt{x^5}} = \frac{3}{8\sqrt{x^5}}$$

$$\therefore \left| \frac{f'''(z)}{6} (x - x_0)(x - x_1)(x - x_2) \right| = \left| \frac{\frac{3}{8\sqrt{z^5}}}{6} (x - 1.2)(x - 1.4)(x - 1.6) \right|$$

$$= \left| \frac{1}{16\sqrt{z^5}} (x - 1.2)(x - 1.4)(x - 1.6) \right| = \left| \frac{1}{16\sqrt{z^5}} (1.5 - 1.2)(1.5 - 1.4)(1.5 - 1.6) \right|$$

$$= \left| \frac{1}{16\sqrt{z^5}} (0.3)(0.1)(-0.1) \right| = \left| \frac{-0.003}{16\sqrt{z^5}} \right|$$

Take  $z = 1.2$  (The smaller value)  $\Rightarrow \left| \frac{-0.003}{16\sqrt{z^5}} \right| \leq \left| \frac{0.003}{16\sqrt{(1.2)^5}} \right|$

$$= \left| \frac{-0.003}{16(1.577)} \right| = \left| \frac{-0.003}{25.232} \right| = \left| -1.188 \times 10^{-4} \right| = 1.188 \times 10^{-4}$$

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